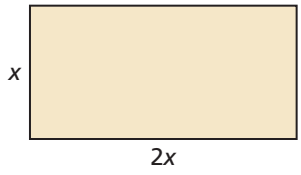
3.2 Linear Functions

Essential Question How can you determine whether a function is linear or nonlinear?

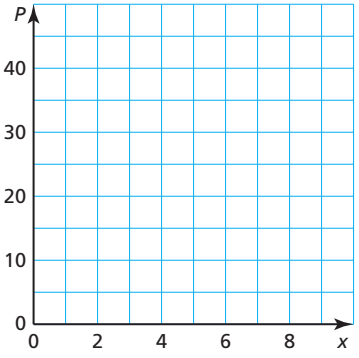
EXPLORATION 1 Finding Patterns for Similar Figures

Work with a partner. Copy and complete each table for the sequence of similar figures. (In parts (a) and (b), use the rectangle shown.) Graph the data in each table. Decide whether each pattern is linear or nonlinear. Justify your conclusion.



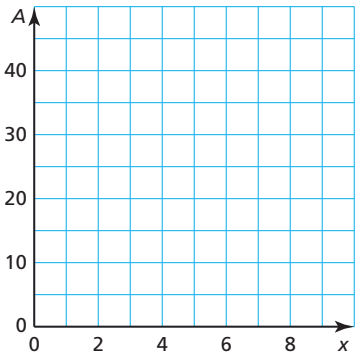
a. perimeters of similar rectangles

x	1	2	3	4	5
P					



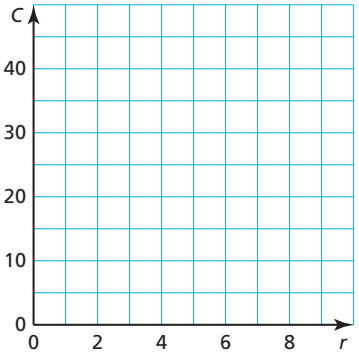
b. areas of similar rectangles

x	1	2	3	4	5
A					



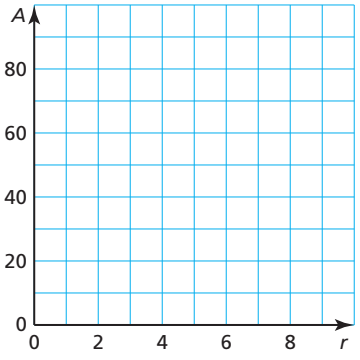
c. circumferences of circles of radius r

r	1	2	3	4	5
C					



d. areas of circles of radius r

r	1	2	3	4	5
A					



USING TOOLS STRATEGICALLY

To be proficient in math, you need to identify relationships using tools, such as tables and graphs.

Communicate Your Answer

2. How do you know that the patterns you found in Exploration 1 represent functions?
3. How can you determine whether a function is linear or nonlinear?
4. Describe two real-life patterns: one that is linear and one that is nonlinear. Use patterns that are different from those described in Exploration 1.

3.2 Lesson

Core Vocabulary

linear equation in two variables, *p. 112*
linear function, *p. 112*
nonlinear function, *p. 112*
solution of a linear equation in two variables, *p. 114*
discrete domain, *p. 114*
continuous domain, *p. 114*

Previous
whole number

What You Will Learn

- Identify linear functions using graphs, tables, and equations.
- Graph linear functions using discrete and continuous data.
- Write real-life problems to fit data.

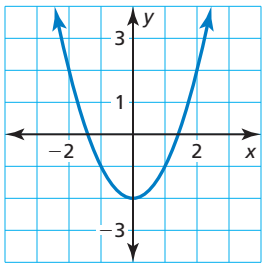
Identifying Linear Functions

A **linear equation in two variables**, x and y, is an equation that can be written in the form $y = mx + b$, where m and b are constants. The graph of a linear equation is a line. Likewise, a **linear function** is a function whose graph is a nonvertical line. A linear function has a constant rate of change and can be represented by a linear equation in two variables. A **nonlinear function** does not have a constant rate of change. So, its graph is *not* a line.

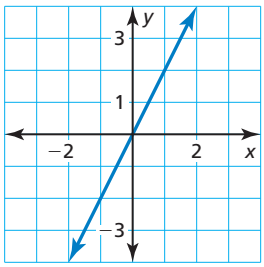
EXAMPLE 1 Identifying Linear Functions Using Graphs

Does the graph represent a *linear* or *nonlinear* function? Explain.

a.



b.



SOLUTION

a. The graph is *not* a line.

▶ So, the function is nonlinear.

b. The graph is a line.

▶ So, the function is linear.

EXAMPLE 2 Identifying Linear Functions Using Tables

Does the table represent a *linear* or *nonlinear* function? Explain.

a.

x	3	6	9	12
y	36	30	24	18

b.

x	1	3	5	7
y	2	9	20	35

SOLUTION

a.

	+3	+3	+3	
x	3	6	9	12
y	36	30	24	18
	−6	−6	−6	

As x increases by 3, y decreases by 6. The rate of change is constant.

▶ So, the function is linear.

b.

	+2	+2	+2	
x	1	3	5	7
y	2	9	20	35
	+7	+11	+15	

As x increases by 2, y increases by different amounts. The rate of change is *not* constant.

▶ So, the function is nonlinear.

REMEMBER

A constant rate of change describes a quantity that changes by equal amounts over equal intervals.

Monitoring Progress

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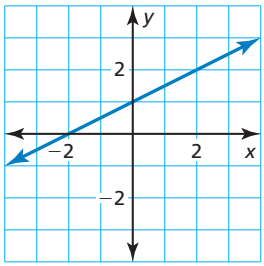
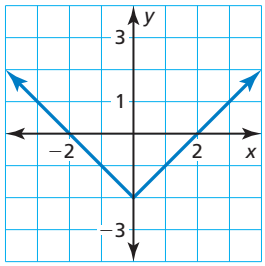
Does the graph or table represent a *linear* or *nonlinear* function? Explain.

1.



2.



3.

x	0	1	2	3
y	3	5	7	9

4.

x	1	2	3	4
y	16	8	4	2

EXAMPLE 3 Identifying Linear Functions Using Equations

Which of the following equations represent linear functions? Explain.

$$y = 3.8,\ y = \sqrt{x},\ y = 3^x,\ y = \frac{2}{x},\ y = 6(x - 1),\ \text{and}\ x^2 - y = 0$$

SOLUTION

You cannot rewrite the equations $y = \sqrt{x}$, $y = 3^x$, $y = \frac{2}{x}$, and $x^2 - y = 0$ in the form $y = mx + b$. So, these equations cannot represent linear functions.

You can rewrite the equation $y = 3.8$ as $y = 0x + 3.8$ and the equation $y = 6(x - 1)$ as $y = 6x - 6$. So, they represent linear functions.

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Does the equation represent a *linear* or *nonlinear* function? Explain.

5. $y = x + 9$

6. $y = \frac{3x}{5}$

7. $y = 5 - 2x^2$

Concept Summary

Representations of Functions

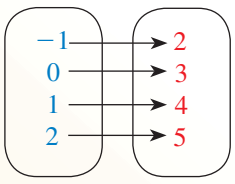
Words An output is 3 more than the input.

Equation $y = x + 3$

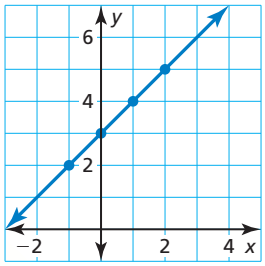
Input-Output Table

Input, x	Output, y
−1	2
0	3
1	4
2	5

Mapping Diagram



Graph



Graphing Linear Functions

A **solution of a linear equation in two variables** is an ordered pair (x, y) that makes the equation true. The graph of a linear equation in two variables is the set of points (x, y) in a coordinate plane that represents all solutions of the equation. Sometimes the points are distinct, and other times the points are connected.

Core Concept

Discrete and Continuous Domains

A **discrete domain** is a set of input values that consists of only certain numbers in an interval.

Example: Integers from 1 to 5

A **continuous domain** is a set of input values that consists of all numbers in an interval.

Example: All numbers from 1 to 5

EXAMPLE 4 Graphing Discrete Data

The linear function $y = 15.95x$ represents the cost y (in dollars) of x tickets for a museum. Each customer can buy a maximum of four tickets.

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

STUDY TIP

The domain of a function depends on the real-life context of the function, not just the equation that represents the function.

SOLUTION

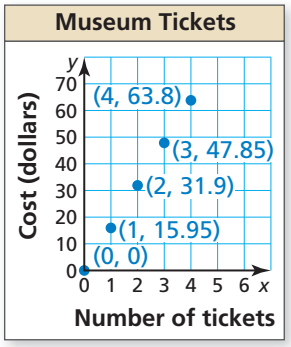
a. You cannot buy part of a ticket, only a certain number of tickets. Because x represents the number of tickets, it must be a whole number. The maximum number of tickets a customer can buy is four.

▶ So, the domain is 0, 1, 2, 3, and 4, and it is discrete.

b. Step 1 Make an input-output table to find the ordered pairs.

Input, x	$15.95x$	Output, y	(x, y)
0	15.95(0)	0	(0, 0)
1	15.95(1)	15.95	(1, 15.95)
2	15.95(2)	31.9	(2, 31.9)
3	15.95(3)	47.85	(3, 47.85)
4	15.95(4)	63.8	(4, 63.8)

Step 2 Plot the ordered pairs. The domain is discrete. So, the graph consists of individual points.



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8. The linear function $m = 50 - 9d$ represents the amount m (in dollars) of money you have after buying d DVDs. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain.

EXAMPLE 5 Graphing Continuous Data

A cereal bar contains 130 calories. The number c of calories consumed is a function of the number b of bars eaten.

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

STUDY TIP

When the domain of a linear function is not specified or cannot be obtained from a real-life context, it is understood to be all real numbers.

SOLUTION

a. As b increases by 1, c increases by 130. The rate of change is constant.

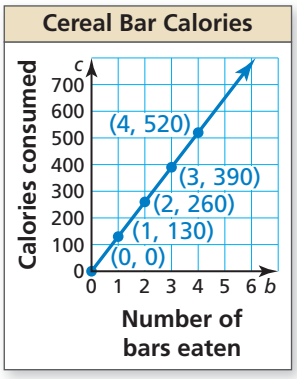
▶ So, this situation represents a linear function.

b. You can eat part of a cereal bar. The number b of bars eaten can be any value greater than or equal to 0.

▶ So, the domain is $b \geq 0$, and it is continuous.

c. **Step 1** Make an input-output table to find ordered pairs.

Input, b	Output, c	(b, c)
0	0	(0, 0)
1	130	(1, 130)
2	260	(2, 260)
3	390	(3, 390)
4	520	(4, 520)



Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points. The line should start at (0, 0) and continue to the right. Use an arrow to indicate that the line continues without end, as shown. The domain is continuous. So, the graph is a line with a domain of $b \geq 0$.

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9. Is the domain discrete or continuous? Explain.

Input Number of stories, x	1	2	3
Output Height of building (feet), y	12	24	36

10. A 20-gallon bathtub is draining at a rate of 2.5 gallons per minute. The number g of gallons remaining is a function of the number m of minutes.

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

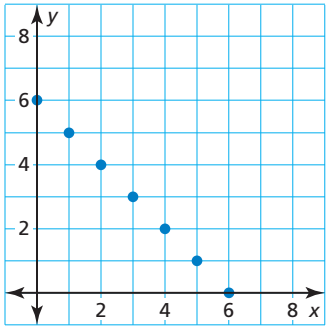
c. Graph the function using its domain.

Writing Real-Life Problems

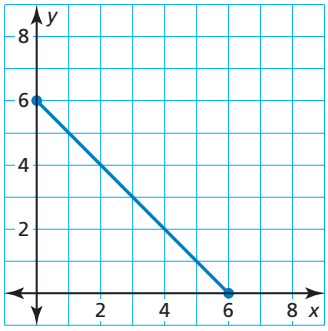
EXAMPLE 6 Writing Real-Life Problems

Write a real-life problem to fit the data shown in each graph. Is the domain of each function *discrete* or *continuous*? Explain.

a.



b.



SOLUTION

a. You want to think of a real-life situation in which there are two variables, x and y. Using the graph, notice that the sum of the variables is always 6, and the value of each variable must be a whole number from 0 to 6.

x	0	1	2	3	4	5	6
y	6	5	4	3	2	1	0

Discrete domain

▶ One possibility is two people bidding against each other on six coins at an auction. Each coin will be purchased by one of the two people. Because it is not possible to purchase part of a coin, the domain is discrete.

b. You want to think of a real-life situation in which there are two variables, x and y. Using the graph, notice that the sum of the variables is always 6, and the value of each variable can be any real number from 0 to 6.

$x + y = 6$ or $y = -x + 6$ Continuous domain

▶ One possibility is two people bidding against each other on 6 ounces of gold dust at an auction. All the dust will be purchased by the two people. Because it is possible to purchase any portion of the dust, the domain is continuous.

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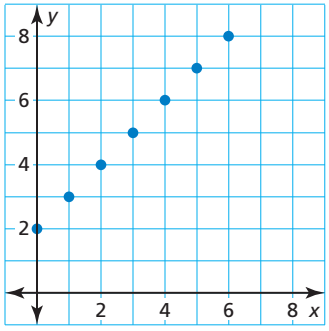
Write a real-life problem to fit the data shown in the graph. Is the domain of the function *discrete* or *continuous*? Explain.

11.



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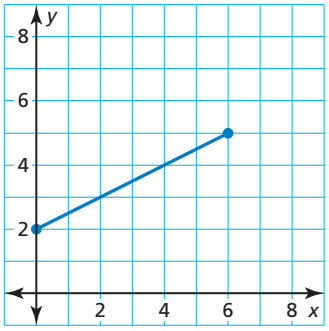


3.2 Exercises

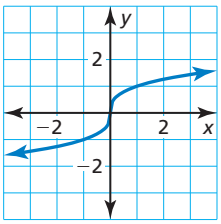
Dynamic Solutions available at *BigIdeasMath.com*

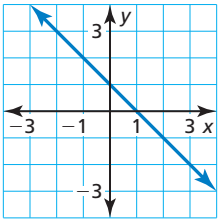
Vocabulary and Core Concept Check

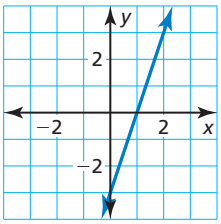
1. **COMPLETE THE SENTENCE** A linear equation in two variables is an equation that can be written in the form _____, where m and b are constants.
2. **VOCABULARY** Compare linear functions and nonlinear functions.
3. **VOCABULARY** Compare discrete domains and continuous domains.
4. **WRITING** How can you tell whether a graph shows a discrete domain or a continuous domain?

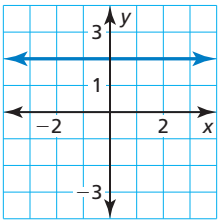
Monitoring Progress and Modeling with Mathematics

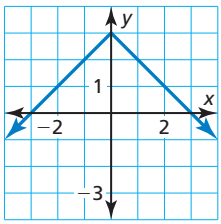
In Exercises 5–10, determine whether the graph represents a *linear* or *nonlinear* function. Explain. *(See Example 1.)*

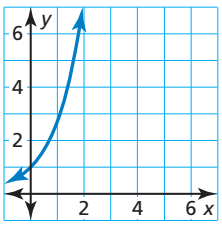
5. 

6. 

7. 

8. 

9. 

10. 

In Exercises 11–14, determine whether the table represents a *linear* or *nonlinear* function. Explain. *(See Example 2.)*

11.

x	1	2	3	4
y	5	10	15	20

12.

x	5	7	9	11
y	−9	−3	−1	3

13.

x	4	8	12	16
y	16	12	7	1

14.

x	−1	0	1	2
y	35	20	5	−10

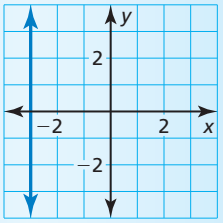
ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in determining whether the table or graph represents a linear function.**

15. ✗

	+2	+2	+2	
x	2	4	6	8
y	4	16	64	256
	×4	×4	×4	

As x increases by 2, y increases by a constant factor of 4. So, the function is linear.

16. ✗



The graph is a line. So, the graph represents a linear function.

In Exercises 17–24, determine whether the equation represents a *linear* or *nonlinear* function. Explain. *(See Example 3.)*

17. $y = x^2 + 13$

18. $y = 7 - 3x$

19. $y = \sqrt[3]{8} - x$

20. $y = 4x(8 - x)$

21. $2 + \frac{1}{6}y = 3x + 4$

22. $y - x = 2x - \frac{2}{3}y$

23. $18x - 2y = 26$

24. $2x + 3y = 9xy$

25. CLASSIFYING FUNCTIONS Which of the following equations *do not* represent linear functions? Explain.

Ⓐ $12 = 2x^2 + 4y^2$

Ⓑ $y - x + 3 = x$

Ⓒ $x = 8$

Ⓓ $x = 9 - \frac{3}{4}y$

Ⓔ $y = \frac{5x}{11}$

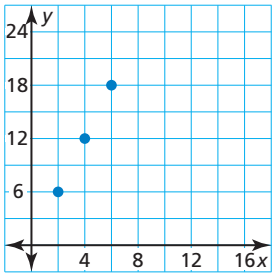
Ⓕ $y = \sqrt{x} + 3$

26. USING STRUCTURE Fill in the table so it represents a linear function.

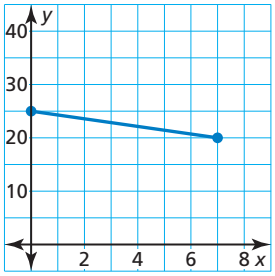
x	5	10	15	20	25
y	−1				11

In Exercises 27 and 28, find the domain of the function represented by the graph. Determine whether the domain is *discrete* or *continuous*. Explain.

27.



28.



In Exercises 29–32, determine whether the domain is *discrete* or *continuous*. Explain.

29.

Input Bags, x	2	4	6
Output Marbles, y	20	40	60

30.

Input Years, x	1	2	3
Output Height of tree (feet), y	6	9	12

31.

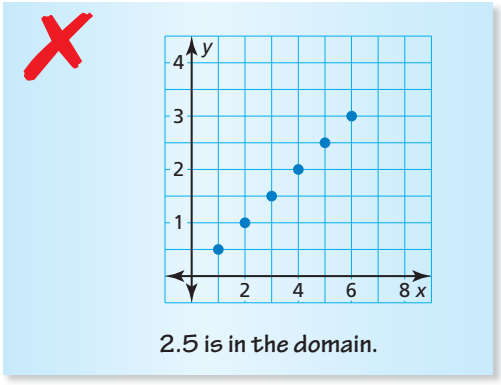
Input Time (hours), x	3	6	9
Output Distance (miles), y	150	300	450

32.

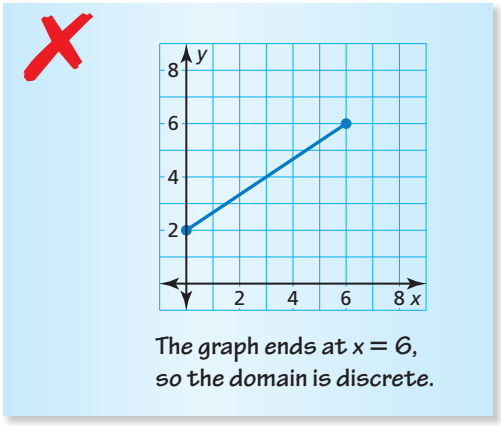
Input Relay teams, x	0	1	2
Output Athletes, y	0	4	8

ERROR ANALYSIS In Exercises 33 and 34, describe and correct the error in the statement about the domain.

33.



34.



35. MODELING WITH MATHEMATICS The linear function $m = 55 - 8.5b$ represents the amount m (in dollars) of money that you have after buying b books. *(See Example 4.)*

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

36. MODELING WITH MATHEMATICS The number y of calories burned after x hours of rock climbing is represented by the linear function $y = 650x$.

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

37. MODELING WITH MATHEMATICS You are researching the speed of sound waves in dry air at 86°F. The table shows the distances d (in miles) sound waves travel in t seconds. *(See Example 5.)*

Time (seconds), t	Distance (miles), d
2	0.434
4	0.868
6	1.302
8	1.736
10	2.170

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

38. MODELING WITH MATHEMATICS The function $y = 30 + 5x$ represents the cost y (in dollars) of having your dog groomed and buying x extra services.

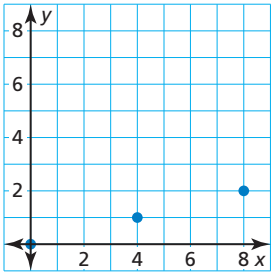
a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

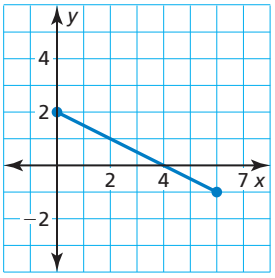
c. Graph the function using its domain.

WRITING In Exercises 39–42, write a real-life problem to fit the data shown in the graph. Determine whether the domain of the function is *discrete* or *continuous*. Explain. *(See Example 6.)*

39.



40.



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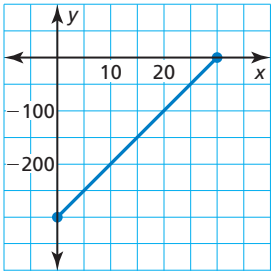

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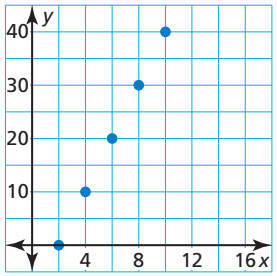


43. USING STRUCTURE The table shows your earnings y (in dollars) for working x hours.

a. What is the missing y-value that makes the table represent a linear function?

b. What is your hourly pay rate?

Time (hours), x	Earnings (dollars), y
4	40.80
5	
6	61.20
7	71.40

44. MAKING AN ARGUMENT The linear function $d = 50t$ represents the distance d (in miles) Car A is from a car rental store after t hours. The table shows the distances Car B is from the rental store.

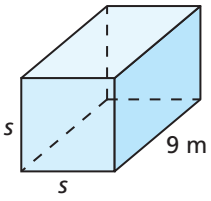
Time (hours), t	Distance (miles), d
1	60
3	180
5	310

a. Does the table represent a linear or nonlinear function? Explain.

b. Your friend claims Car B is moving at a faster rate. Is your friend correct? Explain.

MATHEMATICAL CONNECTIONS **In Exercises 45–48, tell whether the volume of the solid is a linear or nonlinear function of the missing dimension(s). Explain.**

45.



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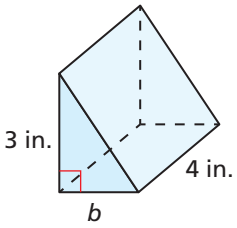

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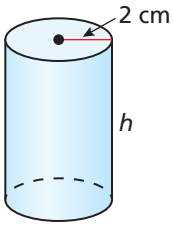

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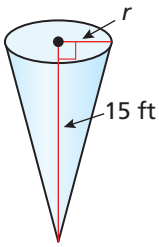


49. **REASONING** A water company fills two different-sized jugs. The first jug can hold x gallons of water. The second jug can hold y gallons of water. The company fills A jugs of the first size and B jugs of the second size. What does each expression represent? Does each expression represent a set of discrete or continuous values?

a. $x + y$

b. $A + B$

c. Ax

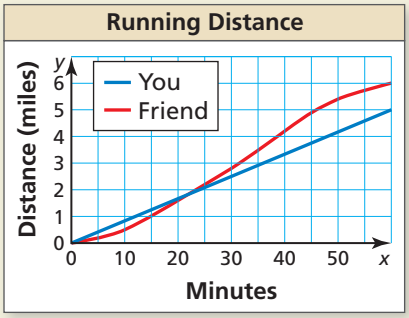
d. $Ax + By$

50. **THOUGHT PROVOKING** You go to a farmer's market to buy tomatoes. Graph a function that represents the cost of buying tomatoes. Explain your reasoning.

51. **CLASSIFYING A FUNCTION** Is the function represented by the ordered pairs linear or nonlinear? Explain your reasoning.

(0, 2), (3, 14), (5, 22), (9, 38), (11, 46)

52. **HOW DO YOU SEE IT?** You and your friend go running. The graph shows the distances you and your friend run.



a. Describe your run and your friend's run. Who runs at a constant rate? How do you know? Why might a person not run at a constant rate?

b. Find the domain of each function. Describe the domains using the context of the problem.

WRITING **In Exercises 53 and 54, describe a real-life situation for the constraints.**

53. The function has at least one negative number in the domain. The domain is continuous.

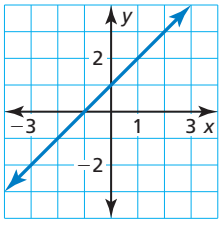
54. The function gives at least one negative number as an output. The domain is discrete.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Tell whether x and y show direct variation. Explain your reasoning. *(Skills Review Handbook)*

55.



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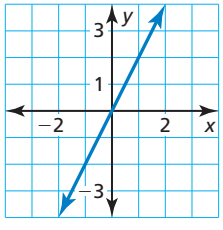

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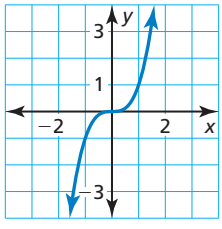

Evaluate the expression when $x = 2$. *(Skills Review Handbook)*

58. $6x + 8$

59. $10 - 2x + 8$

60. $4(x + 2 - 5x)$

61. $\frac{x}{2} + 5x - 7$